ECEN 4413
Automatic Control Systems
Spring 2004
Computer Project

In this project you are going to use the model of an inverted pendulum. It is a benchmark problem in control domain due to its non-linear nature. In order to control the inverted pendulum system you will linearize the system model and use root-locus method to implement a controller. In the end we will try to analyze over all system stability using Simulink/Matlab.

Part 1: Linearization

Below is the non-linear dynamic equation of the inverted pendulum. You need to use this dynamic equation in order to find the linear model of your system. (**friction on the pendulum joint is neglected**)

$$(M+m)\ddot{x}+b\dot{x}+ml\ddot{\theta}\cos\theta - ml\dot{\theta}^2\sin\theta = F$$
$$(I+ml^2)\ddot{\theta}+mgl\sin\theta = -ml\ddot{x}\cos\theta$$

where

M	mass of the cart	0.8 kg
m	mass of the pendulum	0.2 kg
b	friction of the cart	0.3 N/m/sec
l	length to pendulum center of mass	0.5 m
I	inertia of the pendulum	0.003 kg*m^2
F	force applied to the cart	
x	cart position coordinate	
theta	pendulum angle from vertical	

I- Find the equilibrium points of the system. (Two equilibrium points)

II- Linearize system about the equilibrium point. Use $\begin{bmatrix} x & \dot{x} & \theta & \dot{\theta} \end{bmatrix}$ as state variables.

Part 2: Simulation

This part consists of Simulink/Matlab. You are going to implement Simulink model of both linear and non-linear model of the inverted pendulum system and inspect the system response.

I- Implement the inverted pendulum in Simulink environment. Both linear and non-linear model of the system should be in the same ".mdl" file. Plot the system response of the system using the following specifications.

   a. Zero input with initial condition set to $[0\ 0\ 0\ 0]^T$. (zero input zero initial state response)

b. Zero input with initial condition set to $[0\ 0\ 1\ 0]^T$. (zero input response)
c. Step input with initial condition set to $[0\ 0\ 0\ 0]^T$. (zero initial state response)
d. Step input with initial condition set to $[0\ 0\ 1\ 0]^T$.
e. Comment on possible effects of friction for part b. (no modeling or calculation required)

II- Return the plots to part I and specify labels of the output such as X, θ. Find the transfer function of the system from force F pendulum angle θ which is ;

$$\frac{\theta(s)}{U(s)} = \frac{\theta(s)}{F(s)}$$

Part 3: Control

This is the final part of the project. You are going to implement a controller using root-locus method. You are free to play with your model and controller in Simulink. There are no design parameters specified for this part. You need to implement a stable close loop response using root locus plot to get full credit. Following steps will help you to find the controller.

I- Import transfer function that you have found in Part to that is θ (s)/ F (s) into root-locus environment.

II- You can see that system is unstable. Now add a pole zero using interactive environment in order to cancel system zero at the origin

III- Next you will need to add two zeros close to origin somewhere between -1 and -10 on real axisFinally add a far right hand side pole relatively big to the zeros you have located in part III.

IV- Finally you need to play with gain until close loop poles are located in right hand side of the s plane. . (You can use step response plot from analysis menu before you continue to simulink)

V- Hand in the snapshots of the root-locus plot at every stage above.

VI- Implement compensator you have found in the ".mdl" file you have created.

VII- Hand in the response of the system and final ".mdl" file.

Numeric Solution for part 1: You still need to drive this result using linearization. Not doing so may result in lost of some points from the first part. Those who were not able to find this solution can continue on the following sections using the result below.

$$\begin{bmatrix} \dot{x} \\ \ddot{x} \\ \dot{\theta} \\ \ddot{\theta} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & -0.369 & 2.279 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & -0.697 & 22.79 & 0 \end{bmatrix} \begin{bmatrix} x \\ \dot{x} \\ \theta \\ \dot{\theta} \end{bmatrix} + \begin{bmatrix} 0 \\ 1.232 \\ 0 \\ 2.325 \end{bmatrix} F$$

$$\begin{bmatrix} x \\ \theta \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x \\ \dot{x} \\ \theta \\ \dot{\theta} \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} F$$